1.8 Essential Question: HOW DO YOU SOLVE COMPOUND INEQUALITIES?

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words __AND__ or __OR__, the result is called a ___COMPOUND___ ___INEQUALITY___.

$x > -2$ **AND** $x < 4$ can be joined together ___$-2 < x < 4$___
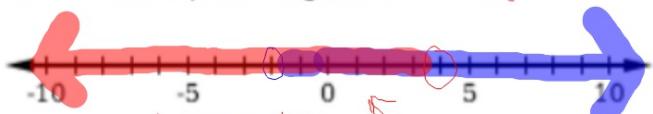
Graph:
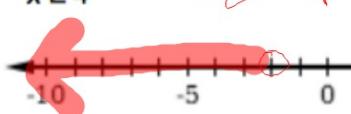
overlap is the answer

$x < -2$ **OR** $x \geq 4$

-2 4

Graph:

No overlap
Go in different directions

Solving Compound Inequalities Involving AND

Example 1: Solve each inequality and graph the solutions.

a) $8 < 3x - 1 \leq 11$
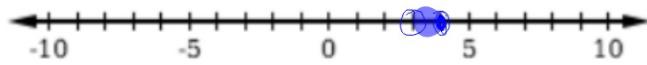
$$+1+1+1$$
$$\overline{}$$
$$\frac{9 < 3x \leq 12}{333}$$

$$3 < x \leq 4$$

b) $-9 < -2x - 11 < -5$

$$+11+11+11$$
$$\overline{}$$
$$\frac{2 < -2x < 6}{-2-2-2}$$

$$-1 > x > -3$$
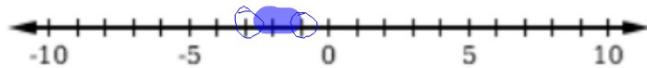
c) $\dfrac{1}{4} < \dfrac{2x-7}{2} < 5$

$$3.75 < x < 8.5$$
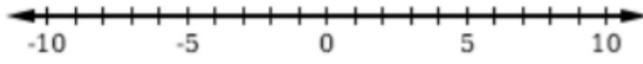
Solving Compound Inequalities Involving OR

*Example 2:* Solve each inequality and graph the solutions.

a) $4x - 2 \leq 18$ OR $3x + 1 > 22$

$\underline{+2 \quad +2} \qquad \underline{-1 \quad -1}$

$\dfrac{4x \leq 20}{4 \quad 4} \qquad \dfrac{3x > 21}{3 \quad 3}$

$x \leq 5 \qquad x > 7$

OR

b) $2 + 5r < 12$ OR $-2r - 5 < -19$

$\underline{-2 \quad -2} \qquad \underline{+5 \quad +5}$

$\dfrac{5r < 10}{5 \quad 5} \qquad \dfrac{-2r < -14}{-2 \quad -2}$
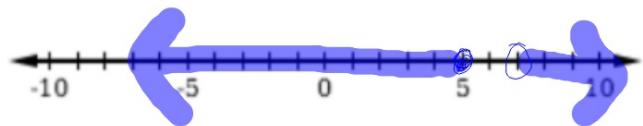
$r < 2 \qquad r > 7$
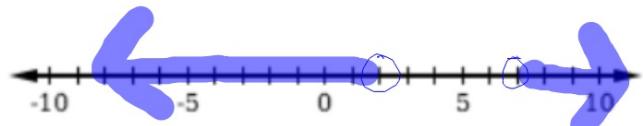
c) $\dfrac{4y + 2}{5} - 5 > 3$ OR $\dfrac{4 - 3y}{6} > 4$

$y > 9.5$ OR $y < 6.6$
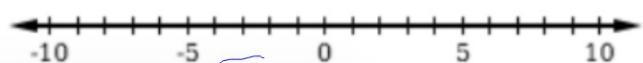
Example 3:

The length of a rectangle is 20 meters longer than the width. The perimeter must be between 80 and 100 meters. What are the possible values for the width of the rectangle?

add sides

AND

$\ell = w + 20$

$80 < \text{perimeter} < 100$

$80 < 2w + 2\ell < 100$

$80 < 2w + 2(w+20) < 100$

$80 < 2w + 2w + 40 < 100$

$80 < 4w + 40 < 100$

$ -40 \phantom{< 4w} -40 \phantom{< 10} -40$

$\dfrac{40}{4} < \dfrac{4w}{4} < \dfrac{60}{4}$

$\boxed{10 < w < 15}$

Summary: