1.14 General Number: Recognizing Patterns

Explore

1. Consider this sequence of numbers: 2, −4, 8, −16, 32
x−2 x−2 x−2

Fill in the next three numbers in the sequence based on a pattern you observe. Write the rule you used to generate the numbers. Repeat this at least two more times by considering a different pattern in the numbers.

	Rule used
a. 2, −4, −10, −16, −22	−6
b. 2, −4, −16, −256, −65536	squarE and opposite
c. 2, −4, 2, −4, 2	repeat 2, −4

Discover

It is human nature to generalize patterns that we see. This is an especially useful skill in mathematics. It will often be necessary in this course for you to write an expression to represent a pattern as you try to model a situation. This section will focus on a couple of particularly useful types of patterns and on how to generalize patterns and write expressions to represent them.

As you work through this section, think about these questions:

1. Can you look at a sequence of numbers and explain the pattern?
2. Are you able to express a pattern mathematically?
3. What are two different types of reasoning that are required in mathematics?

We will often need to look at a list of numbers and make a guess about the pattern that exists. This skill can help us create models, graphs, and projections for the future. The process of making an educated guess or conclusion based on the evidence provided is known as forming a **conjecture.** You will need to make a conjecture in the next problem in order to write more terms of a sequence.

2. The Fibonacci sequence is a famous sequence of numbers that follows an interesting pattern. Write the next six numbers in the sequence and describe the rule you are using to generate those values.

+ = + =
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …
+ = + =

A sequence begins with the first term, followed by the second term, and so on. The *n*th term refers to the term in the "*n*th" position and is a generic expression that shows the general form of each term.

3. Determine the pattern in the following sequence and write an expression for the *n*th term. It may help to label the term number for each term to see the relationship between the term number and the values in the term.

$\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}, \ldots$ $\boxed{\frac{n+2}{n+1}}$

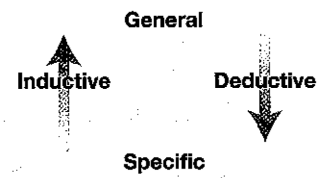
Term #
$n = 1, 2, 3, 4, 5$

A particular type of reasoning, inductive reasoning, was needed to generalize the pattern in the sequence in #3.

LOOK IT UP

Types of Reasoning

When we generalize and move from a specific case to the general one, the process is known as **inductive reasoning**. When we move from the general rule to a specific case, we are using **deductive reasoning**.



Since we use both types of reasoning so often in our daily lives, we often have trouble distinguishing them. Inductive reasoning is used to form **hypotheses** or **conjectures**. Deductive reasoning is used to prove conjectures.

We can make conjectures based on what we have seen, but the conjectures may not be accurate for a variety of reasons. A conjecture can be proven true using deductive reasoning or proven false with a counterexample. A counterexample is simply an example that does not follow the conjecture and therefore shows that the conjecture is not true.

FOR EXAMPLE, you might conjecture that squaring a number always produces a positive result, since $1^2 = 1$, $2^2 = 4$, and so on. However, $0^2 = 0$, which is not positive. This counterexample disproves the conjecture because it shows a case in which squaring a number does not produce a positive result.

It takes only one counterexample to disprove a conjecture. However, a conjecture cannot be proven even with multiple examples. Instead, a conjecture can be proven only with deductive logic.

EXAMPLE

A student knows that a number is even if the last digit in the number is even. So she wonders if a number must be divisible by 3 if the last digit is divisible by 3. Find a counterexample that proves her conjecture is false.

SOLUTION

The student's conjecture is that if the last digit is divisible by 3, then the number is also divisible by 3. So a counterexample would be a number with a last digit that is divisible by 3 but that is not itself divisible by 3. Try the first two-digit number that could possibly work as a counterexample: 13. This number has a last digit (3) that is divisible by 3, but the number 13 is not divisible by 3. We do not get a whole-number answer when we divide 13 by 3, so 13 serves as a counterexample.

Using inductive reasoning to make conjectures is helpful, but it does have its limits, since some conjectures turn out to be false, as we saw in the last example. If we are making conjectures to determine the pattern in a sequence, our job will be a lot easier if we can identify when we have particular types of sequences. There are two types of sequences that are common and are important for other topics in this course.

LOOK IT UP

Sequences

Subtract (+ neg)

In an **arithmetic sequence**, the next term is always found by adding the same number to the previous term. (first term) + (n − 1)·(adder) $t_1 + (n-1)d$

FOR EXAMPLE, −8, −6, −4, −2, 0, 2, . . . is an arithmetic sequence because each term is found by adding 2 to the previous term.

In a **geometric sequence**, the next term is always found by multiplying the previous term by the same number.

division ($* \frac{1}{\#}$)

FOR EXAMPLE, −8, 16, −32, 64, −128, . . . is a geometric sequence because each term is found by multiplying the previous term by −2. $(\text{first term})\cdot(\text{multiplier})^{n-1}$

$t_1(r)^{n-1}$

Consider the sequence −17, −12, −7, −2, ... Notice that each term can be found by adding 5 to the previous term, which makes the sequence an arithmetic sequence. Each term requires one more 5 than the previous term. Specifically,

$$\begin{array}{ll} & \text{1st Term} \\ \text{2nd term:}\ -12 & = -17 + 5 = -17 + 1(5) \\ \text{3rd term:}\ -7 & = -17 + 5 + 5 = -17 + 2(5) \\ \text{4th term:}\ -2 & = -17 + 5 + 5 + 5 = -17 + 3(5) \end{array}$$

Each term can be found by starting with the first term, −17, and adding a certain number of 5's. The number of 5's added to get each term is always *one fewer* than the term number. So, if the term number is called n, then the number of 5's is $n - 1$. This is a key insight if we want to write an expression for a generic nth term in the sequence. The nth term can be found using the formula

$$n\text{th term: } -17 + (n-1)5$$

Remember?
Division can always be rewritten as multiplication.

4. Consider this sequence: 100, 50, 25, 12.5, . . .

a. Describe how each term is generated from the previous term. Is this sequence arithmetic, geometric, or neither?

÷2 geometric

b. Complete the following table:

Term #	Term	Calculation to get term from the first term
1	100	________________
2	50	$100(\frac{1}{2})^1$
3	25	$100(\frac{1}{2})(\frac{1}{2}) = 100(\frac{1}{2})^2$
4	12.5	$100(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = 100(\frac{1}{2})^3$
5	6.25	$100(\frac{1}{2})^4$
6	3.125	$100(\frac{1}{2})^5$

term # is always 1 more than exponent

c. Write an expression for the nth term.

$100\left(\frac{1}{2}\right)^{n-1}$

5. Identify each of the following sequences as arithmetic, geometric, or neither. If the sequence is arithmetic or geometric, write an expression for the nth term.

a. 6, 18, 54, 162, ... G, $6(3)^{n-1}$

b. 1, 2, 4, 7, 11, ... N

c. −16, −20, −24, −28, ... A, $-16+(n-1)\cdot(-4)$

Sometimes it is easier to write an expression for the nth term of a sequence if you use the "0th term" instead of the 1st term that's shown in the sequence. That is, you can use the term that would come before the first term.

Consider the arithmetic sequence that begins 1, 3, 5, 7, ... If we write a formula for the nth term based on the first term of 1, we get $1 + (n - 1)2$.

Another option is to work backward and determine what number should come before the 1 in the pattern. If each number is found by adding 2 to the previous number, then we can work the sequence backward by subtracting 2. So the "0th term" that would come before the 1st term is $1 - 2 = -1$.

We can then use this "0th term" to write an expression for the nth term in the sequence. Notice that, if we start with −1, the number of 2's needed is always the same as the term number.

		0th Term
1st term:	$1 =$	$-1 + 2 = -1 + 1(2)$
2nd term:	$3 =$	$-1 + 2 + 2 = -1 + 2(2)$
3rd term:	$5 =$	$-1 + 2 + 2 + 2 = -1 + 3(2)$
4th term:	$7 =$	$-1 + 2 + 2 + 2 + 2 = -1 + 4(2)$

If we write a formula for the nth term based on the first term of 1, we get $-1 + 2n$. This formula is equivalent to the first formula we found and will generate the same terms, but it looks simpler.

Connect

You can't always identify a sequence from the first couple of terms. It might be possible to generate more than one sequence from the first two terms.

6. Consider the following start of a sequence: 2, 10, ...

a. Write the next five terms of the sequence if it's arithmetic. Write an expression for the nth term using the 1st term of the sequence and another expression using the 0th term.

2, 10, 18, 26, 34, 42, 50

+8 +8

$2+(n-1)\cdot 8$

b. Write the next five terms of the sequence if it's geometric. Write an expression for the nth term using the 1st term of the sequence and another expression using the 0th term.

2, 10, 50, 250, 1250, 6250, 31250

*5 *5

$2(5)^{n-1}$

Reflect

WRAP-UP

What's the point?

Identifying and generalizing patterns are powerful skills that can require the use of both inductive and deductive reasoning. Learning particular patterns, like arithmetic and geometric, that occur in life and nature can be particularly useful.

What did you learn?

How to make conjectures and generalize patterns
How to identify and use arithmetic and geometric sequences

1.14 Homework

Skills MyMathLab

It is fine to use the MyMathLab help aids *View an Example* and *Help Me Solve This*, but make sure you are not dependent on them. To be successful in the course, you need to know how to start and finish a problem.

First complete the MyMathLab homework online. Then work the two exercises to check your understanding.

1. Find the pattern in each sequence and use it to list the next two terms.

   a. 5, 17, 29, 41,

   b. 18, 14, 10, 6,

   c. −9, 4, −8, 5, −7, 6,

2. What is the next term of the sequence?
   $3x, 4x + 1, 5x + 2, 6x + 3,$