2.6 Does Order Matter?: Rewriting Expressions

Explore

1. Imagine that you are making purchases at a department store and you see that there is a 10%-off sale. You also have an online coupon for 5% off your purchase.

   a. Is the final price affected by the order in which the discounts are applied? Why or why not?

   (10%)

   $\$60 \cdot 0.10 = 6 \quad 60 - 6 = \$54$

   (5%)

   $\$54 \cdot 0.05 = 2.7 \quad 54 - 2.7 = \boxed{\$51.30}$

   $\$60 \cdot .05 = 3 \quad 60 - 3 = 57$

   $\$57 \cdot .10 = 5.7 \quad 57 - 5.7 = \$51.30$

   order doesn't matter

   b. Will applying the 10% discount and then the 5% discount result in a 15% discount? Explain why or why not.

   $60 \cdot 0.15 = 9 \quad 60 - 9 = 51$

   NO

Discover

As we saw with order of operations in Section 2.5, the order in which multiple operations are done usually makes a difference. However, there are mathematical calculations in which order does not matter. This section will help you consider when and where order might not matter in mathematics.

By the end of the section, you should be able to answer the following questions:

1. For which operations does order not matter?
2. How can mathematical properties be used to make mental math easier to do?

In the *Explore*, the order of the discounts does not make a difference because we found the amounts by applying two multipliers. When you multiply numbers, the order of the factors does not affect the final product. Similarly, when you add numbers, the order of those numbers does not affect the final sum.

The commutative property states that, for some operations, the order of the numbers involved does not matter. For example, imagine your commute to work. You travel from home to work and then from work to home. The order changes, but the trip is the same length (provided you use the same route).

LOOK IT UP

Commutative Properties

The **commutative properties** of addition and multiplication say that we can change the order in which we multiply or add numbers without changing the result. The word "commute" means to exchange or interchange two things.

Suppose a and b are real numbers.

$$a + b = b + a \quad \text{(commutative property of addition)}$$
$$ab = ba \quad \text{(commutative property of multiplication)}$$

EXAMPLES:

$$4 + 5 = 5 + 4$$
$$4(5) = 5(4)$$
$$x + 2y = 2y + x$$
$$xy = yx$$

The commutative properties are often used when you first learn the addition and multiplication facts in order to reduce the number of facts that you have to memorize. Knowing that 3×4 is the same as 4×3 reduces two facts to one. Likewise, $2 + 9$ is the same as $9 + 2$, and it is easier to count up 2 from 9 than to count up 9 from 2.

2. Use the commutative properties to rewrite each expression and then combine like terms.

a. $ab + ba = ab + ab = 2ab$

b. $ab - ba = ab - ab = 0$

c. $3x + 4y - 5x + 8y = 3x - 5x + 4y + 8y = \boxed{-2x + 12y}$

The commutative properties tell us that the order does not matter when we are performing addition and multiplication. Are there operations for which order *does* matter? When we are performing subtraction and division, the order of the numbers does affect the answer. For example, $12 \div 6 = 2$ but $6 \div 12 = \frac{1}{2}$.

3. We know that $4 - 11$ and $11 - 4$ are not equivalent expressions, but sometimes we need to reverse the order of the numbers in an expression involving subtraction. For the following expressions, write the terms in a different order, using only the commutative property for addition.

a. $x + 2 = 2 + x$

b. $4 - 11 = 4 + (-11) = -11 + 4$

c. $x - 2 = -2 + x$

d. $2 - x = -x + 2$

To summarize, if we are performing addition or multiplication, we can reverse numbers without affecting the result. If we are performing subtraction, we can still reverse the numbers, but we need to keep the appropriate sign with each number.

The commutative property says that we can change the order when adding or multiplying. Another property, the associative property, says that terms can be regrouped when we add or multiply.

LOOK IT UP

Associative Properties

The **associative properties** of addition and multiplication say that we can change the grouping when we multiply or add three or more numbers without changing the result.

Suppose a, b, and c are real numbers.

$$a + (b + c) = (a + b) + c \quad \text{(associative property of addition)}$$
$$a(bc) = (ab)c \quad \text{(associative property of multiplication)}$$

Sometimes the associative property is applied in order to group compatible numbers together.

FOR EXAMPLE, instead of $15 + (5 + 17)$, we can apply the associative property to get $(15 + 5) + 17$. This allows the addition of 15 and 5 to be performed first, since it is easy to add those numbers mentally. We get $20 + 17$, which is 37.

Similarly, the expression $(26 \cdot 5) \cdot 2$ can be written as $26 \cdot (5 \cdot 2)$ to make the calculations easier to perform. We get 26(10), which is 260.

Remember that the commutative property involves changing order, and the associative property involves regrouping.

Sometimes we combine the commutative property with the associative property to make it easier to perform calculations mentally.

4. Use the stated property to rewrite the computation to make it easier to compute mentally, and then find the result.

a. $17 + (13 + 68)$ associative property $(17+13)+68$ $30+68$ $=90$

b. $(8 \cdot 15)(2)$ associative property $8 \cdot (15 \cdot 2) = 8 \cdot 30 = 240$

c. $19 + (72 + 11)$ commutative property and then associative property

$19+(11+72)$

$(19+11)+72 = (30)+72 = 102$

Connect

To add and subtract polynomials, you can first use the commutative and associative properties to rearrange the terms. Suppose we want to add the following trinomials. We need to reorder and regroup the terms to get like terms together.

$$\begin{aligned}&(5x^4 + 8x^3 - 2x) + (x^4 - 2x^3 - 5x)\\ &= (5x^4 + x^4) + (8x^3 - 2x^3) + (-2x - 5x)\\ &= 6x^4 + 6x^3 - 7x\end{aligned}$$

5. Add or subtract as indicated.

a. $(3x^2 - 5x + 6) + (4x^2 - 7x - 5) = 7x^2 - 12x + 1$

b. $(5y - 14) + (7y^2 - 8y + 3) = 7y^2 - 3y - 11$

c. $(9x^2 + 6x + 15) - 4x^2 = 5x^2 + 6x + 15$